A Copula Approach to Reliability Analysis of Dependent Consecutive k-out-of-n:G Systems

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Abstract This paper presents a novel reliability estimation method for dependent consecutive k-out-of-n:G systems, leveraging multivariate copula theory. In such systems, the entire system functions if and only if at least k consecutive components are operational. Classical reliability models often rely on the assumption of independence between components an assumption that fails to reflect the true behavior of many real-world systems. The proposed method leverages the flexibility of copulas to model complex dependency structures among component lifetimes, independently of their marginal distributions. This copula-based framework is specifically tailored to the structure of consecutive k-out-of-n systems and introduces a new methodology for estimating system reliability under dependence. The models are implemented in MATLAB. Results confirm the potential of the copula-based approach to improve reliability assessment in complex systems where component interdependence plays a critical role.

Keywords Functional data · Nonparametric estimation · Rate of Convergence · Uniform almost-complete convergence.

Mathematics Subject Classification (2010) 62N05 68M15 68M20 90B25

Keywords consecutive k-out-of-n: G system, estimation multivariate copulas, MATLAB.

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1 Introduction

The reliability assessment of engineering systems is a critical concern across various industrial sectors such as telecommunications, energy, and transportation. Among the most frequently studied configurations, consecutive k-out-of-n: G systems are particularly important: a system is considered operational if at least k consecutive components function properly. This setup is especially relevant in real-world applications such as pipeline networks, communication links, or sensor arrays, where the failure of adjacent units can severely compromise overall system performance.

Traditional models often assume statistical independence between component states. While this simplifies the analysis, it fails to capture realistic dependencies caused by shared environments, operational stress, or common failure mechanisms. Research on systems with dependent and non-identically distributed components remains relatively limited. Notable contributions include [1] for parallel structures and [2] for general coherent systems. [3] studied time-dependent reliability of linear and circular consecutive systems with independent but heterogeneous components, while [4] analyzed residual lifetimes under similar conditions. More recently, [5] and [6] established reliability bounds for coherent systems with arbitrarily distributed elements.

The use of copula-based approaches has proven valuable for modeling dependence in reliability analysis. For instance, [7] derived exact reliability expressions for systems equipped with cold standby components, using copulas to capture marginal dependence. [8] incorporated degradation and shock processes into the modeling of k-out-of-n:F systems with component interactions. [9] applied Clayton and Gumbel copulas to capture load-sharing dependence in consecutive configurations. [10] estimated performance for non-identical components based on wind speed data, and [11] introduced a robust estimation method for one-shot devices subject to correlated failures.

In addition to these contributions, other recent efforts have addressed performance evaluation under dependence and heterogeneity. [12] proposed a copula framework for multi-state, non-repairable systems. [13] offered exact analytical formulations for cold standby systems, while [14] modeled three-state structures using non-homogeneous Markov dependence.

This study introduces a novel estimation method for the reliability of k-out-of-n systems with consecutive dependence, leveraging multivariate copula theory. This approach enables flexible modeling of complex interaction patterns among components, without requiring restrictive assumptions on marginal distributions. A range of copula families including Gaussian, Gumbel, Clayton, and Frank are used to accommodate various types of dependence. The methodology is implemented in MATLAB, enabling efficient simulation and performance evaluation.

While previous research has highlighted the growing role of copulas in system modeling, the approach proposed here distinguishes itself by providing a unified, flexible, and accurate estimation framework, specifically tailored to systems with consecutive interdependencies. This work represents a significant step forward in the realistic modeling and analysis of engineering system reliability.

The remainder of this paper is structured as follows. In Section 2, we propose a novel approach for estimating the reliability of consecutive consecutive k-out-of-n: G systems using multivariate copulas, allowing for flexible modeling of dependence between component lifetimes. Section 3 presents a detailed graphical and numerical analysis comparing analytical results with copula-based estimations, using several families of copulas to assess their effectiveness. Section 4 concludes the study by summarizing the main findings.

Notation 1 *n: number of components in the system.*
k : the minimum number of consecutive components required for the system to be good.
T_i *: lifetime of component i,* $i = 1, \ldots, n$.
$F_i(t) = P(T_i \leq t)$*: distribution function of* T_i.
$R_{k/n:G}(t)$*: reliability of a consecutive k-out-of-n: G system.*
$C(F_{T_{i_1}}, \cdots, F_{T_{i_{k+1}}})$ *: a multivariate copula.*
$\hat{C}(F_{T_{i_1}}, \cdots, F_{T_{i_l}})$ *: estimation of multivariate copula.*
$\hat{\theta}_i^{PMV}$*: Pseudo-likelihood estimator of* θ_i.
$\hat{R}_{k/n:G}(t)$*: Pseudo-likelihood estimator of* $R_{k/n:G}(t)$.

2 consecutive k out of n: G system

In this section, we propose a novel method for estimating the reliability of a consecutive k-out-of-n: G system using multivariate copula theory. Unlike classical approaches that assume independence among components, our method explicitly accounts for the dependence structure between component lifetimes, independent of their marginal distributions. The system's reliability is first expressed in terms of a copula function that models this dependence, and the copula is then estimated using suitable statistical techniques, enabling a comprehensive representation of the joint behavior of the components.

Definition 1 *A consecutive k-out-of-n: G system consists of n components, this system works if and only if at least k consecutive components works*

Proposition 1 *: For* $2k \geq n$*, the reliability of a consecutive k-out-of-n: G system with dependent components is defined as follows:*

$$R_{k/n:G}(t) = (n-k+1)P[T_1 > t, T_2 > t, ..., T_k > t] - (n-k)P[T_1 > t, T_2 > t, ..., T_{k+1} > t]. \quad (1)$$

Which can reformulated by using Copula theory as:

$$R_{k/n:G}(t) = 1 - \sum_{l=1}^{k} (-1)^{l-1} \sum_{1 \leq i_1 \cdots i_l \leq k} C(F_{T_{i_1}}, \cdots, F_{T_{i_l}}) + (n-k)(-1)^k \sum_{1 \leq i_1 \cdots i_l \leq k+1} C(F_{T_{i_1}}, \cdots, F_{T_{i_{k+1}}}). \quad (2)$$

Where $C(F_{T_{i_1}}, \cdots, F_{T_{i_{k+1}}})$ *denotes a multivariate copula.*

Proposition 2 *: The reliability of a consecutive k-out-of-n:G system can be estimated using estimation of multivariate copula* $\hat{C}(F_{T_{i_1}},\cdots,F_{T_{i_l}})$ *such as the Gaussian, Clayton, Student, or Frank copula.*

$$R_{k/n:G}(t) = 1-\sum_{l=1}^{k}(-1)^{l-1}\sum_{1\le i_1\cdots i_l\le k}\hat{C}(F_{T_{i_1}},\cdots,F_{T_{i_l}})+(n-k)(-1)^{k}\sum_{1\le i_1\cdots i_l\le k+1}\hat{C}(F_{T_{i_1}},\cdots,F_{T_{i_{k+1}}}). \tag{3}$$

Proposition 3 *To estimate the reliability of a consecutive k-out-of-n:G system, the parameter θ is determined by maximizing the pseudo log-likelihood function, which captures the dependence structure among component lifetimes:*

$$L(\theta) = \sum_{i=1}^{n} lnC_{\theta}(\hat{F}_{1:n}(x_{i_1}),\cdots,\hat{F}_{d:n}(x_{i_d}))$$

The estimator $\hat{\theta}_i^{PMV}$ *is defined by*

$$\hat{\theta}_{i_l}^{PMV} = argmaxL_{i_l}^{*}(\theta)$$

let be

$$L_{i_l}^{*}(\theta) = \sum_{1\le i_1\cdots i_l\le k} lnC(\hat{F}_{T_{i_1}},\cdots,\hat{F}_{T_{i_l}}). \tag{4}$$

so

$$\hat{R}_{k/n:G}(t) = 1-\sum_{l=1}^{k}(-1)^{l-1}\sum_{1\le i_1\cdots i_l\le k}\hat{\theta}_{i_l}^{PMV}+(n-k)(-1)^{k}\hat{\theta}_{i_{k+1}}^{PMV}. \tag{5}$$

Proof (Proof of Proposition 1) For $2k \ge n$, the reliability of a consecutive k-out-of-n:G system with dependent components is defined as follows:

$$R_{k/n:G}(t) = (n-k+1)P[T_1>t,T_2>t,...,T_k>t]-(n-k)P[T_1>t,T_2>t,...,T_{k+1}>t]. \tag{6}$$

Such that

$$P(\bigcap_{i=1}^{k}T_i>t) = 1-P(\bigcup_{i=1}^{k}(T_i\le t))$$

$$=1-\sum_{l=1}^{k}(-1)^{l-1}\sum_{1\le_{i_1}<i_2<\cdots<i_l\le_k}P[(T_{i_1}\le t)\bigcap\cdots\bigcap(T_{i_l}\le t)]$$

and

$$P(\bigcap_{i=1}^{k+1}T_i>t) = 1-\sum_{l=1}^{k+1}(-1)^{l-1}\sum_{1\le_{i_1}<i_2<\cdots<i_l\le_{k+1}}P[(T_{i_1}\le t)\bigcap\cdots\bigcap(T_{i_l}\le t)]$$

using that

$$C(F_{T_{i_1}},\cdots,F_{T_{i_l}}) = P[(T_{i_1}\le t)\bigcap\cdots\bigcap(T_{i_l}\le t)]$$

so

$$R_{k/n:G}(t) = (n-k+1)[1-\sum_{l=1}^{k}(-1)^{l-1}\sum_{1\le_{i_1}<i_2<\cdots<i_l\le_k}P[(T_{i_1}\le t)\bigcap\cdots\bigcap(T_{i_l}\le t)]]$$

$$-(n-k)[1-\sum_{l=1}^{k+1}(-1)^{l-1}\sum_{1\leq i_1<i_2<\cdots<i_l\leq k+1}P[(T_{i_1}\leq t)\bigcap\cdots\bigcap(T_{i_l}\leq t)]$$

$$R_{k/n:G}(t)=(n-k+1)-(n-k+1)\sum_{l=1}^{k}(-1)^{l-1}\sum_{1\leq i_1<i_2<\cdots<i_l\leq k}P[(T_{i_1}\leq t)\bigcap\cdots\bigcap(T_{i_l}\leq t)]]$$

$$-(n-k)+(n-k)\sum_{l=1}^{k+1}(-1)^{l-1}\sum_{1\leq i_1<i_2<\cdots<i_l\leq k+1}P[(T_{i_1}\leq t)\bigcap\cdots\bigcap(T_{i_l}\leq t)]$$

$$R_{k/n:G}(t)=1-(n-k)\sum_{l=1}^{k}(-1)^{l-1}\sum_{1\leq i_1<i_2<\cdots<i_l\leq k}P[(T_{i_1}\leq t)\bigcap\cdots\bigcap(T_{i_l}\leq t)]]$$

$$-\sum_{l=1}^{k}(-1)^{l-1}\sum_{1\leq i_1<i_2<\cdots<i_l\leq k}P[(T_{i_1}\leq t)\bigcap\cdots\bigcap(T_{i_l}\leq t)]]$$

$$+(n-k)\sum_{l=1}^{k}(-1)^{l-1}\sum_{1\leq i_1<i_2<\cdots<i_l\leq k+1}P[(T_{i_1}\leq t)\bigcap\cdots\bigcap(T_{i_l}\leq t)]$$

$$+(n-k)(-1)^{k}\sum_{1\leq i_1<i_2<\cdots<i_l\leq k+1}P[(T_{i_1}\leq t)\bigcap\cdots\bigcap(T_{i_l}\leq t)]$$

$$R_{k/n:G}(t)=1-\sum_{l=1}^{k}(-1)^{l-1}\sum_{1\leq i_1<i_2<\cdots<i_l\leq k}P[(T_{i_1}\leq t)\bigcap\cdots\bigcap(T_{i_l}\leq t)]]$$

$$+(n-k)(-1)^{k}\sum_{1\leq i_1<i_2<\cdots<i_l\leq k+1}P[(T_{i_1}\leq t)\bigcap\cdots\bigcap(T_{i_l}\leq t)]$$

$$R_{k/n:G}(t)=1-\sum_{l=1}^{k}(-1)^{l-1}\sum_{1\leq i_1\cdots i_l\leq k}C(F_{T_{i_1}},\cdots,F_{T_{i_l}})+(n-k)(-1)^{k}\sum_{1\leq i_1\cdots i_l\leq k+1}C(F_{T_{i_1}},\cdots,F_{T_{i_{k+1}}})$$

Proof (Proof of Proposition 3) The consecutive k-out-of-n: G system can be estimated by maximizing the pseudo log-likelihood to estimate θ with

$$L(\theta)=\sum_{i=1}^{n}lnC_{\theta}(\hat{F}_{1:n}(x_{i_1}),\cdots,\hat{F}_{d:n}(x_{i_d}))$$

such that l'estimateur $\hat{\theta}_n^{PMV}$ defined by

$$\hat{\theta}_{i_l}^{PMV}=argmaxL_{i_l}^{*}(\theta)$$

let be

$$L_{i_l}^{*}(\theta)=\sum_{1\leq i_1\cdots i_l\leq k}lnC(\hat{F}_{T_{i_1}},\cdots,\hat{F}_{T_{i_l}})$$

and when

$$R_{k/n:G}(t)=1-\sum_{l=1}^{k}(-1)^{l-1}\sum_{1\leq i_1\cdots i_l\leq k}C(F_{T_{i_1}},\cdots,F_{T_{i_l}})+(n-k)(-1)^{k}\sum_{1\leq i_1\cdots i_l\leq k+1}C(F_{T_{i_1}},\cdots,F_{T_{i_{k+1}}})$$

so we can estimate $R_{k/n:G}(t)$ by

$$\hat{R}_{k/n:G}(t)=1-\sum_{l=1}^{k}(-1)^{l}\sum_{1\leq i_1\cdots i_l\leq k}\hat{\theta}_{i_l}^{PMV}+(n-k)(-1)^{k}\sum_{1\leq i_1\cdots i_l\leq k+1}\hat{\theta}_{i_{k+1}}^{PMV}$$

3 Graphical Analysis of the Copula-Based Estimation: Exact vs. Empirical

This section presents a graphical and numerical comparison between analytical reliability values and those estimated using various copula-based methods. The analysis evaluates the performance of different copulas Gaussian, Student-t , Clayton, and Gumbel using both plots and tabulated results. By comparing exact and estimated reliability values, we aim to assess the accuracy and robustness of the copula-based estimation techniques. The results, illustrated through figures and summarized in tables, provide insight into the relative performance of each copula in modeling multivariate dependence, particularly when applied to systems with transformed exponential margins.
To facilitate the interpretation of the results, we propose expressing the system reliability function $R_{k/n:G}(t)$ in the following form:

$$R_{k/n:G}(t) = A + B$$

where

$$A = 1 - \sum_{l=1}^{k} (-1)^{l-1} \sum_{1 \le i_1 \cdots i_l \le k} C(F_{T_{i_1}}, \cdots, F_{T_{i_l}})$$

and

$$B = (n-k)(-1)^k \sum_{1 \le i_1 \cdots i_l \le k+1} C(F_{T_{i_1}}, \cdots, F_{T_{i_{k+1}}})$$

Case 1: Let us consider a consecutive 1-out-of-3:G system. The system reliability, calculated using Equation (3), is plotted in Figure 1 using the Gaussian copula.

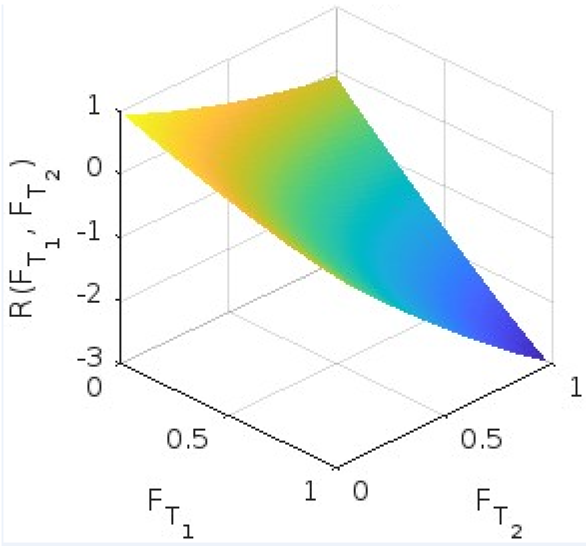

Fig. 1 Calculation of the exact reliability value of dependent 1-out-of-3:G systems using Gaussian copula.

Case 2: Let us consider a consecutive 1-out-of-3:G system. The reliability, computed using Equation (3), is plotted in Figure 1 using the Clayton copula.

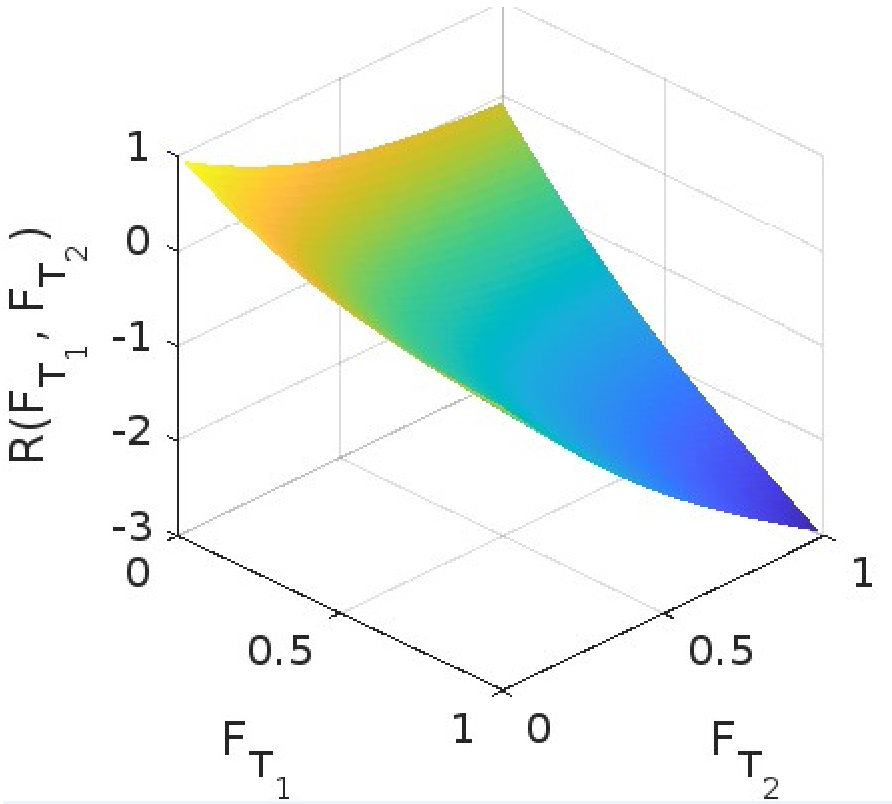

Fig. 2 Calculation of the exact reliability of dependent 1-out-of-3:G systems using Clayton copula.

Case 3: Let us consider a consecutive 1-out-of-3:G system. The exact values of reliability and the estimated values using Gaussian copula are plotted in Figure 3.

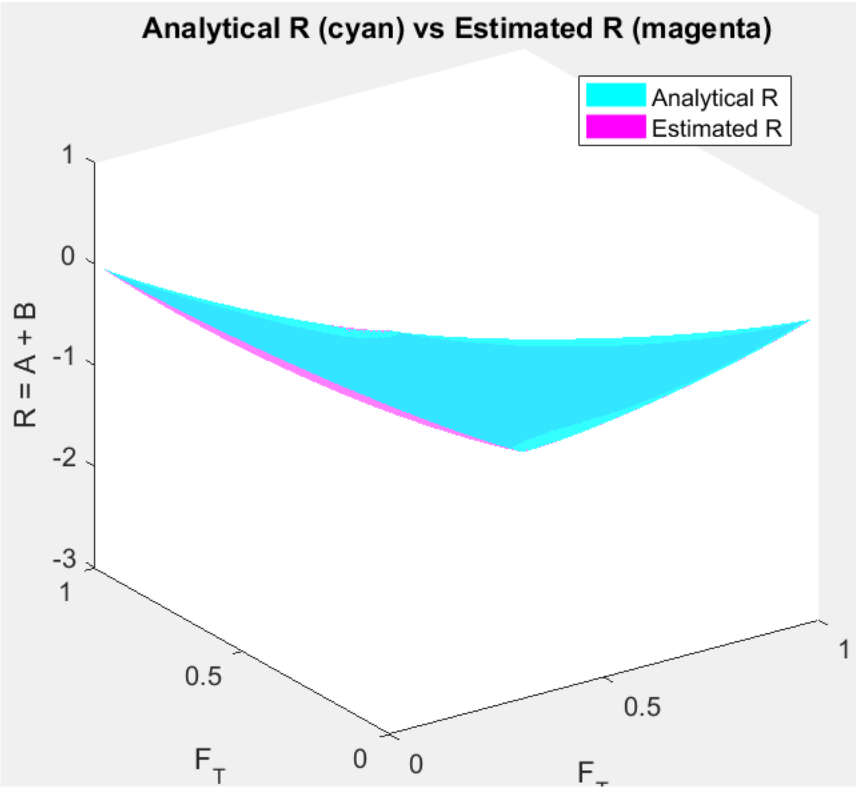

Fig. 3 Exact and estimated values of the reliability of dependent 1-out-of-3:G systems using Gaussian copula.

Case 4: Let us consider a consecutive 2-out-of-5:G system. The exact values of reliability and the estimated values using the Gaussian copula are plotted in Figure 4.

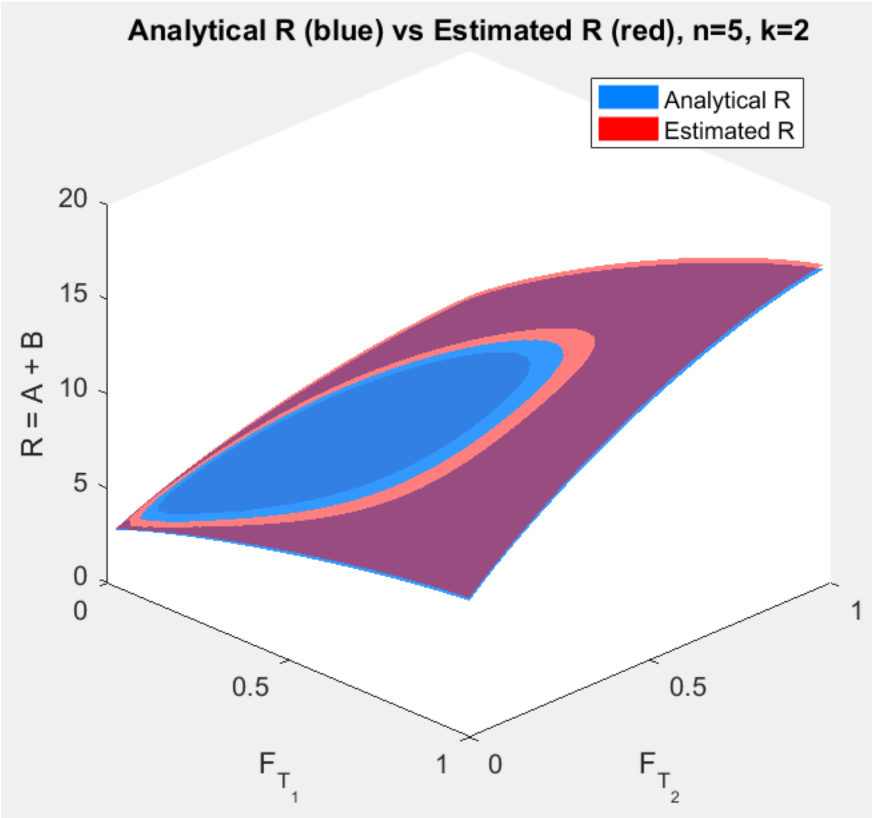

Fig. 4 Difference between the exact and estimated values of the reliability of dependent 2-out-of-5:G systems using Gaussian copula.

Case 5: Let us consider a consecutive 2-out-of-5:G system. The exact values of reliability and the estimated values using Student copulas are plotted in Figure 5.

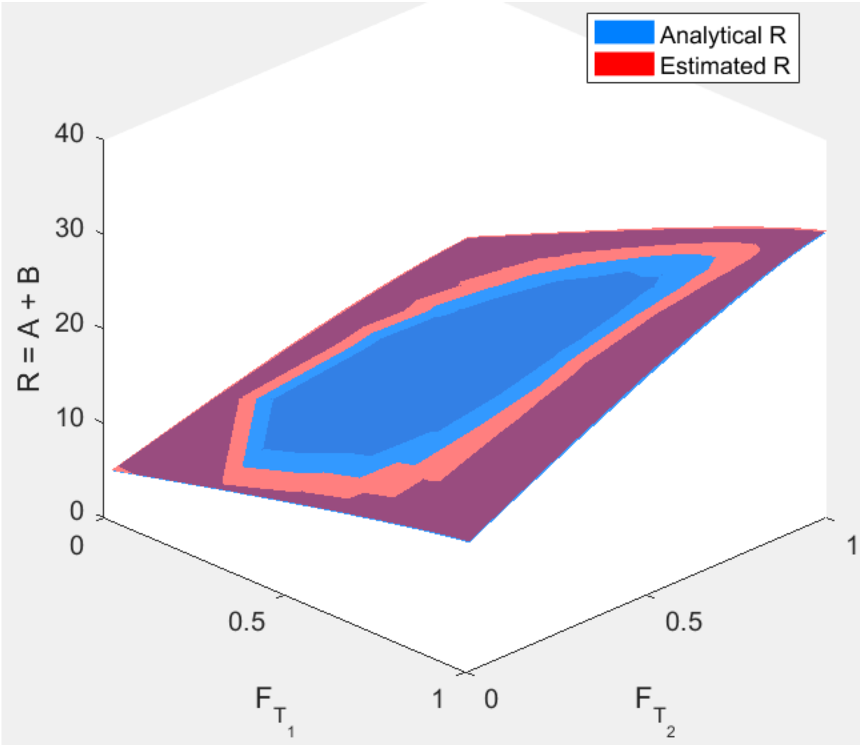

Fig. 5 Difference between the exact and estimated values of the reliability of dependent 2-out-of-5:G systems using Student copula.

The following tables provide a numerical comparison between analytical and estimated values of the dependence measure R for multivariate distributions. The estimates were obtained using copula-based simulations with transformed exponential marginals.

Each row corresponds to a simulated data point, where $F_1(t)$ to $F_5(t)$ are the values of the marginal distributions. The columns labeled R_{Analytic} and $R_{\text{Estimated}}$ represent the theoretical and empirical values of the dependence measure, respectively. The absolute difference between these two values is also reported.

These results are shown for two different copulas: the Gaussian copula and the Gumbel copula. The small discrepancies observed confirm the high accuracy and robustness of the estimation method across different configurations.

$F_1(t)$	$F_2(t)$	$F_3(t)$	$F_4(t)$	$F_5(t)$	$R_{Analytic}$	$R_{Estimed}$	$\text{Diff}_a bs$
0.741	0.192	0.050	0.100	0.675	3.6885	3.7011	0.0126
0.056	0.812	0.291	0.221	0.950	5.8491	5.8460	0.0032
0.887	0.313	0.886	0.295	0.870	13.7953	13.8210	0.0258
0.566	0.425	0.236	0.279	0.050	4.2050	4.2162	0.0111
0.474	0.369	0.661	0.559	0.705	12.3834	12.4103	0.0269
0.176	0.612	0.658	0.856	0.050	6.5724	6.5866	0.0142
0.191	0.332	0.283	0.050	0.149	2.6122	2.6236	0.0114
0.562	0.581	0.950	0.066	0.950	11.3460	11.3492	0.0033
0.133	0.912	0.724	0.211	0.808	9.0931	9.0978	0.0046
0.221	0.548	0.483	0.396	0.338	7.0496	7.0735	0.0239

Table 1 Table of differences: Analytical vs. Estimated R ($n = 5$,$k = 2$, $F_i \sim Exp(i)$, Gaussien Copula

The examples and graphical illustrations reveal the following observations:

- The absolute differences between $R_{Analytic}$ and R_{Est} are very small
- Relative differences remain very low , confirming that the method provides a very accurate approximation, even for transformed exponential marginals. Such small discrepancies are expected due to the randomness inherent in simulations.
- This table highlights the strong performance of the analytical model and its coherence with the simulated data. The method using copulas combined with exponential marginals provides reliable estimates of dependence in a multivariate context. The minor differences observed between analytical and estimated values are expected and reflect the accuracy of both calculation and simulation.
- The small gap between $R_{Analytic}$ and R_{Est} indicates that the Gaussian copula and the empirical estimation approach are well suited to modeling dependence among these transformed exponential variables.

$F_1(t)$	$F_2(t)$	$F_3(t)$	$F_4(t)$	$F_5(t)$	$R_{Analytic}$	$R_{Estimed}$	$\text{Diff}_a bs$
0.269	0.800	0.895	0.668	0.405	5.2245	5.2140	0.0105
0.133	0.885	0.731	0.703	0.775	5.6166	5.6158	0.0008
0.406	0.423	0.338	0.598	0.405	3.9657	3.9559	0.0098
0.878	0.093	0.107	0.762	0.050	2.3178	2.3126	0.0053
0.134	0.720	0.723	0.067	0.713	3.5032	3.5028	0.0004
0.422	0.602	0.843	0.794	0.402	5.5242	5.5258	0.0017
0.950	0.714	0.670	0.152	0.465	4.8654	4.8600	0.0053
0.421	0.249	0.479	0.927	0.529	4.3292	4.3241	0.0051
0.736	0.375	0.193	0.144	0.723	3.3491	3.3543	0.0051

Table 2 Table of differences: Analytical vs. Estimated R ($n = 5$, $k = 3$, $F_i \sim Exp(i)$, Gumbel Copula

- The absolute differences are all very small (less than 0.02), indicating that the estimation method closely reproduces the analytical value. The relative differences are also low. This confirms the robustness of the estimation.
- Whether the F_i values are low or high, the accuracy remains stable. For example, row 5 with $F_5 = 0.05$ and row 3 with several F_i values close to 0.7 to 0.9 show similarly very small deviations.
- The estimation method is validated for the Gumbel copula with the tested parameters.
- The small relative error suggests that the numerical approximation is adequate.
- These results confirm the relevance of using this copula in multivariate dependence studies, particularly with transformed exponential margins.

4 Performance Comparison of Copula Models via Relative Error

The following boxplot illustrates the distribution of relative errors % across different copula models.

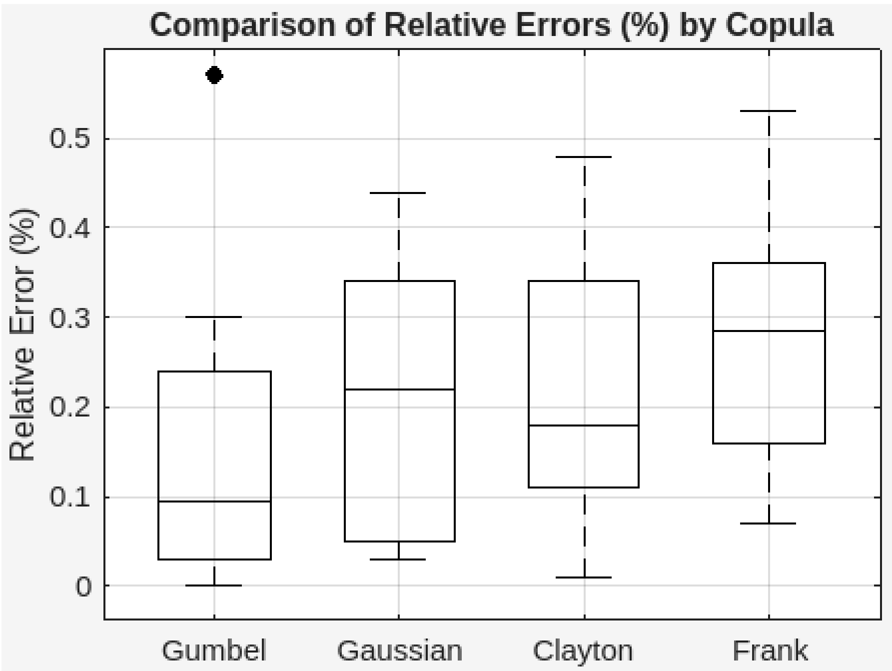

Fig. 6 Comparison of relative errors % according to the copula.

- The analysis of relative errors % for each copula-Gumbel, Gaussian, Clayton, and Frank-reveals clear differences in performance.

- Gumbel copula shows the lowest and most stable errors, indicating the best fit to the data. It performs particularly well when upper-tail dependence is present.
- Gaussian copula performs moderately, with some low errors but greater variability. Its limitation to model only symmetric, linear dependence might explain the less consistent performance.
- Clayton copula shows mixed results, with a few low errors but also several higher ones. It may work better when lower-tail dependence dominates but is overall less reliable.
- Frank copula has the highest variability and generally higher errors, suggesting it is the least suited for the data in this case, likely due to its lack of tail dependence modeling.

Remark 1 $R = A + B$ *The primary objectives of this construction are to:*
Provide a summary measure of bivariate dependence between two distributions $R = A + B$ *is recomputed to reflect the estimated dependence structure. The computed values from the true and estimated copulas are then compared both graphically and numerically. This comparison illustrates the accuracy of the estimation procedure and highlights the effects of finite-sample variability on the captured dependence structure.*

5 Conclusion

The main outcomes derived from this work can be summarized as follows:

- A new reliability formula for dependent consecutive k-out-of-n:G systems has been developed, leveraging multivariate copula theory to model inter-component dependencies.
- A pseudo log-likelihood maximization approach has been proposed for the estimation of the copula parameters, enabling practical application of the model to real data.
- The methodology allows for flexible modeling using various families of copulas, including Gaussian, Clayton, and Student-t copulas, depending on the dependence structure of the system.
- Graphical comparisons between copula-based estimations and empirical reliability results reveal the accuracy and robustness of the proposed method, especially in capturing dependency effects.

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