Prime Labeling in Duplicate Structures of Select Graphs

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ABSTRACT

A graph G with n vertices is said to admit prime labeling if its vertices can be labeled with distinct positive integers not exceeding n such that the labels of each pair of adjacent vertices are relatively prime. A graph G which admits prime labeling is called a prime graph. In this paper we prove that the duplicate graph of the path P_n *where* $n \geq 2$, the duplicate graph of cycle C_n *where* $n \geq 3$, the duplicate graph of star S_n *where* $n \geq 3$, the duplicate graph of double star $DS_{n,n}$ *where* $n \geq 2$, the duplicate graph of comb graph Cb_n *where* $n \geq 2$, and the duplicate graph of bistar graph $B_{n,n}$ for all integers $n \geq 2$ are prime labeling.

Keywords: *Graph labeling, Prime labeling, Prime graph, Duplicate graph*

I. INTRODUCTION

The concept of graph labeling was introduced by Rosa in 1967[4]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges), then the labeling is called a vertex labeling (or edge labeling). In the intervening years various labeling of graphs have been investigated in over 2000 papers. The notion of prime labeling graph was introduced by Roger Entringer and was discussed in a paper by tout [11]. Fu and Huang [3] proved that P_n and $K_{1,n}$ are prime graphs. Lee et al [5] proved that W_n is a prime graph if and only if n is even. Deretsky et al [2] proved that C_n is a prime graph. Meena and Kavitha proved the prime labeling for some butterfly related graph and also investigated the prime labeling of duplication of some star related graphs [6] [7].

The concept of duplicate graph was introduced by E. Sampathkumar and he proved many result on it [8]. K. Thirusangu, P.P. Ulaganathan and B. Selvam have proved that the duplicate graph of a path graph P_m is cordial[9]. K. Thirusangu, P.P Ulaganathan and P. Vijayakumar proved that the duplicate graph of ladder graph is cordial, total cordial and prime cordial [10].In this paper we consider only simple finite, Undirected and nontrivial graph $G = (V(G), E(G))$ with the vertex set $V(G)$ and the edge set $E(G)$. The set of vertices adjacent to a vertex u of G is denoted by $N(u)$. For notation and terminology we refer to Bondy and Murthy [1].

II. PRELIMINARIES

Definition 2.1A graph labeling is an assignment of integers to the vertices or edges or both subject to the certain condition(s). If the domain of the mapping is the set of vertices (or edges), then the labeling is called a vertex labeling (or an edge labeling).

Definition 2.2 A prime labeling of a graph G of order n is an injective function $f : V \to \{1, 2, 3, ...n\}$ such that for every pair of adjacent vertices u and v, $\gcd(f(u), f(v)) = 1$.The graph which admits prime labeling is called prime graph.

Definition 2.3 Let $G(V,E)$ be a simple graph. A duplicate graph of G is $DG=(V_1,E_1)$ where the vertex set $V_1=V\cup V^1$ and $V\cap V^1=\phi$ and $f:V\to V^1$ is bijective (for $v\in V$, we write $f(v)=V^1$) and the edge set E_1 of DG is defined as: The edge uv is in E if and only if both uv^1 and u^1v are edges in E_1.

Definition 2.4Double star $DS_{n,n}$ is a tree $K_{1,n,n}$ obtained from the star $K_{1,n}$ by adding the new pendant edge of the existing n pendant vertices. It has $2n+1$ vertices and $2n$ edges.

Definition 2.5The comb C_{bn} is obtained from a path P_n by attaching a pendant edge at each vertex of the path P_n.

Definition 2.6Abistar $B_{n,n}$ is a graph obtained from K_2 by joining n pendant edges to each end of K_2. The edge K_2 is called the central edge of $B_{n,n}$ and the vertices of K_2 are called the central vertices of $B_{n,n}$

Bertrand's Postulate 2.7For every positive integer $n>1$ there is a prime p such that $n<p<2n.$

In this paper we prove that the duplicate graph of the path P_n $where\, n\geq 2,$ the duplicate graph of cycle C_n $where\, n\geq 3,$ the duplicate graph of star S_n $where\, n\geq 3,$ the duplicate graph of double star $DS_{n,n}$ $where\, n\geq 2,$ the duplicate graph of comb graph Cb_n $where\, n\geq 2,$ and the duplicate graph ofbistar graph $B_{n,n}$ where $n\geq 2$ are prime labeling.

III. MAIN RESULTS

Theorem 3.1 The duplicate graph of path P_n for all integers $n\geq 2$ isprime.

Proof

$Let\ V(P_n)=\{v_i\,/\,1\leq i\leq n\}\ and\ E(\mathrm{P}_n)=\{v_iv_{i+1}\,/\,1\leq i\leq n-1\}$

Let G be the duplicate graph of path P_n and let $v_1,v_2,v_3,...,v_n,v_1^1,v_2^1,v_3^1,...,v_n^1$ be the new vertices and the edges are $e_1,\mathrm{e}_2,\mathrm{e}_3,...,\mathrm{e}_{n-1},e_1^1,\mathrm{e}_2^1,\mathrm{e}_3^1,...,\mathrm{e}_{n-1}^1$ respectively.

Then $V(G)=\{v_i,v_i^1\,/\,1\leq i\leq n\}$

$\mathrm{E}(G)=\{v_iv_{i+1}^1\,/\,1\leq i\leq n-1\}\cup\{v_i^1v_{i+1}\,/\,1\leq i\leq n-1\}$.

Now $|V(G)|=2n, |\mathrm{E}(G)|=2(n-1)$.

Define a labeling $f:V(G)\to\{1,2,3,...,2n\}$ as follows

let $f(v_i)=i$ if $i=\begin{cases}1,3,5,...,n-1\ for\ n\ is\, even\\ or\\ 1,3,5,...,n\end{cases}$

$f(v_i)=n+i$ if $i=\begin{cases}2,4,6,...,n\ for\ 'n'\ is\, even\\ or\\ 2,4,6,...,n-1\end{cases}$

$f(v_i^1)=n+i$ if $i=\begin{cases}1,3,5,...,n-1\ for\ n\ is\, even\\ or\\ 1,3,5,...,n\end{cases}$

$f(v_i^1) = i$ if $i = \begin{cases} 2,4,6,\dots,n \text{ for } 'n' \text{ is even} \\ or \\ 2,4,6,\dots,n-1 \end{cases}$

Clearly all the labels are distinct.

For any edge $e = v_i v_{i+1}^1 \in G,$

$\gcd(f(v_i), f(v_{i+1}^1)) = \gcd(i, i+1)$ if $i = \begin{cases} 1,3,5,\dots,n-1 \text{ for } n \text{ is even} \\ or \\ 1,3,5,\dots,n-2 \end{cases}$

$=1$ Since the labels are consecutive integers.

$\gcd(f(v_i), f(v_{i+1}^1)) = \gcd(\text{n}+i, \text{n}+i+1)$ if $i = \begin{cases} 2,4,6,\dots,n-2 \text{ for } 'n' \text{ is even} \\ or \\ 2,4,6,\dots,n-1 \end{cases}$

$=1$ Since the labels are consecutive integers.

For any edge $e = v_i^1 v_{i+1} \in G$,

$\gcd(f(v_i^1), f(v_{i+1})) = \gcd(\text{n}+i, \text{n}+i+1)$ if $i = \begin{cases} 1,3,5,\dots,n-1 \text{ for } n \text{ is even} \\ or \\ 1,3,5,\dots,n-2 \end{cases}$

$=1$ Since the labels are consecutive integers.

$\gcd(f(v_i^1), f(v_{i+1})) = \gcd(i, i+1)$ if $i = \begin{cases} 2,4,6,\dots,n-2 \text{ for } 'n' \text{ is even} \\ or \\ 2,4,6,\dots,n-1 \end{cases}$

$=1$ Since they are all consecutive positive integers.

Thus f admits prime labeling.

Hence duplicate graph of path P_n is a prime graph.

Theorem 3.2 The duplicate graph of cycle C_n for all integers $n \geq 3$ is a prime graph.

Proof

$Let\ V(\text{G}) = \{v_i / 1 \leq i \leq n\}$

$E(\text{G}) = \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{v_n v_1\}.$

Let G be the duplicate graph obtained by duplicate the cycle C_n.

Let $v_1, v_2, v_3, \dots, v_n, v_1^1, v_2^1, v_3^1, \dots, v_n^1$ and $\text{e}_1, \text{e}_2, \text{e}_3, \dots, \text{e}_n$, $\text{e}_1^1, \text{e}_2^1, \text{e}_3^1, \dots, \text{e}_n^1$ respectively be the new set of vertices and edges of the duplicate graph of cycle C_n.

$Then\ V(\text{G}) = \{v_i v_i^1 / 1 \leq i \leq n\}$

$E(\text{G}) = \left\{v_i v_{i+1}^1 / 1 \leq i \leq n-1\right\} \cup \left\{v_1 v_n^1\right\} \cup \left\{v_i^1 v_{i+1} / 1 \leq i \leq n-1\right\} \cup \left\{v_1^1 v_n\right\}.$

Now $|V(G)| = 2n$ and $E(G) = 2n$.

Define a labeling $f: V(G) \to \{1, 2, 3, \dots, 2n\}$ as follows.

Let $f(v_i) = i+1$ if $i = \begin{cases} 1,3,5,\dots,n-1 \; for\; n\; is\, even \\ or \\ 1,3,5,\dots,n \end{cases}$

$f(v_i) = n+i+1$ if $i = \begin{cases} 2,4,6,\dots,n-2 \; for\; n\; is\, even \\ or \\ 2,4,6,\dots,n-1 \end{cases}$

$If\ 'n'\ is\ even\ then\ f(v_n) = 1.$

$f(v_i^1) = n+i+1$ if $i = \begin{cases} 1,3,5,\dots,n-1 \; for\; n\; is\, even \\ or \\ 1,3,5,\dots,n-2 \end{cases}$

$f(v_i^1) = i+1$ if $i = \begin{cases} 2,4,6,\dots,n \; for\; n\; is\, even \\ or \\ 2,4,6,\dots,n-1 \end{cases}$

$If\ 'n'\ is\ odd\ then\ f(v_n^1) = 1.$

Clearly vertex label are distinct.

For any edge $e = v_i v_{i+1}^1 \in E(G)$,

$\gcd(f(v_i), f(v_{i+1}^1)) = \gcd(i+1, i+2)$ if $i = \begin{cases} 1,3,5,\dots,n-1 \; for\; n\; is\, even \\ or \\ 1,3,5,\dots,n-2 \end{cases}$

$= 1$ Since they are consecutive integers.

$\gcd(f(v_i), f(v_{i+1}^1)) = \gcd(\mathrm{n}+i+1, \mathrm{n}+i+2)$ if $i = \begin{cases} 2,4,6,\dots,n-2 \; for\; 'n'\; is\, even \\ or \\ 2,4,6,\dots,n-1 \end{cases}$

$= 1$ Since these are consecutive integers.

For any edge $e = v_i^1 v_{i+1} \in G$,

$\gcd(f(v_i^1), f(v_{i+1})) = \gcd(\mathrm{n}+i+1, \mathrm{n}+i+2)$ if $i = \begin{cases} 1,3,5,\dots,n-3 \; for\; n\; is\, even \\ or \\ 1,3,5,\dots,n-2 \end{cases}$

$= 1$ Since they are consecutive positive integers.

$\gcd(f(v_i^1), f(v_{i+1})) = \gcd(i+1, i+2)$ if $i = \begin{cases} 2,4,6,\dots,n-2 \; for\; 'n'\; is\, even \\ or \\ 2,4,6,\dots,n-1 \end{cases}$

$= 1$ Since the vertex labels are consecutive integers.

For any edge $e = v_1 v_n^1 \in G,$

$\gcd(f(v_1), f(v_n^1)) = \gcd(2, \mathrm{odd}) = 1$

and for any edge $e = v_1^1 v_n \in G$, If $'n'$ is even then $\gcd(f(v_1^1), f(v_n)) = \gcd(\mathrm{n}+i+1, 1) = 1$.

Similarly If $'n'$ is odd then $\gcd(f(v_1^1), f(v_n)) = \gcd(\mathrm{n}+i+1, n+i) = 1$. Since both are consecutive integers.

Now f admits prime labeling. Hence G is the prime graph.

Theorem 3.3 The duplicate graph of star S_n, for all integers $n \geq 3$ is a prime graph.

Proof

Let $V(S_n)=\{v_i\ /\ 0\le i\le n\}$ *and* $E(S_n)=\{v_0v_i\ /\ 1\le i\le n\}$.

Let G be the duplicate graph of the star S_n and let $v_0,v_1,...,v_n,v_0^1,v_1^1,...,v_n^1$ be the new vertices and $e_1^1,e_2^1,...,e_n^1,e_1,e_2,...,e_n$ be the new edges of the duplicate graph of star S_n.

Then $V(G)=\{v_i,v_i^1\setminus 0\le i\le n\}$,

$E(G)=\{v_0v_i^1\setminus 1\le i\le n\}\cup\{v_0^1v_i\setminus 1\le i\le n\}$.

Then $|V(G)|=2(n+1)$ and $|E(G)|=2n$.

Define the labeling $f:V(G)\to\{1,2,3,...,2(n+1)\}$ as follows

Let $f(v_i)=2i+1$ $\quad$ *if* $i=0,1,2,3,...n$

$f(v_i^1)=2i+2$ $\quad$ *if* $i=0,1,2,3,...n$

Clearly all the labels are distinct.

For any edge $e=v_0v_i^1\in G$,

$\gcd(f(v_0),f(v_i^1))=\gcd(1,2(i+1))$ if $i=1,2,3,...n$

$=1$

and for any edge $e=v_0^1v_i\in G$,

$\gcd(f(v_0^1),f(v_i))=\gcd(2,2i+1)$ if $i=1,2,3,...n$

$=\gcd(2,odd)$ if $i=1,2,3,...n$

$=1$.

Thus f admits prime labeling.

Hence the duplicate graph of the star S_n is a prime graph.

Theorem 3.4 The duplicate graph of double star $DS_{n,n}$ for all integers $n\ge 2$ is a prime graph.

Proof

Let $V(DS_{n,n})=\{v_i\ /\ 0\le i\le 2n\}$

$E(DS_{n,n})=\{v_0v_i\ /\ 1\le i\le n\}\cup\{v_iv_{n+i}\ /\ 1\le i\le n\}$.

Let G be the duplicate graph of double star $DS_{n,n}$, $n\ge 2$.

So we get $v_0,v_1,...,v_n,v_{n+1},...,v_{2n}$, $v_0^1,v_1^1,...,v_n^1,v_{n+1}^1,...,v_{2n}^1$ and $e_1,e_2,...,e_n,e_{n+1},...,e_{2n},e_1^1,e_2^1,...,e_n^1,e_{n+1}^1,...,e_{2n}^1$ respectively be the new set of vertices and edgesof the duplicate graph of double star $DS_{n,n}$.

Now $V(G)=\{v_iv_i^1\ /\ 0\le i\le 2n\}$ and

$E(G)=\{v_0v_i^1\ /\ 1\le i\le n\}\cup\{v_0^1v_i\ /\ 1\le i\le n\}\cup\{v_iv_{n+i}^1\ /\ 1\le i\le n\}\cup\{v_i^1v_{n+i}\ /\ 1\le i\le n\}$ then $|V(G)|=4n+2$ and $|E(G)|=4n$.

Define the labeling $f: V(G) \rightarrow \{1,2,3,...,4n+2\}$ as follows

Let $f(v_i) = 2i+1$ if $i = 0,1,2,3,...2n$

$f(v_0^1) = 2$

$f(v_i^1) = 2n + 2(i+1)$ if $i = 1,2,3,...n$

$f(v_i^1) = 2(i+1) - 2n$ if $i = n+1, n+2, n+3, ...2n$

Clearly all the labels are distinct.

For any edge $e = v_0 v_i^1 \in G$,

$\gcd(f(v_0), f(v_i^1)) = \gcd(1, f(v_i^1)) = 1$ if $i = 1,2,3,...n$

For any edge $e = v_0^1 v_i \in G,$

$\gcd(f(v_0^1), f(v_i)) = \gcd(2, f(v_i))$ if $i = 1,2,3,...n$

$= \gcd(2, odd)$ if $i = 1,2,3,...n$

$= 1$

For any edge $e = v_i v_{n+i}^1 \in G,$

$\gcd(f(v_i), f(v_{n+i}^1)) = \gcd(2i+1, 2i+2)$ if $i = 1,2,3,...n$

$= 1$ since the vertex labels are consecutive integers.

For any edge $e = v_i^1 v_{n+i} \in G$,

$\gcd(f(v_i^1), f(v_{n+i})) = \gcd(2n + 2(i+1), 2n + 2i + 1)$ if $i = 1,2,3,...n$

$= 1$. Since they are consecutive positive integers.

Thus f admits prime labeling.

Hence the duplicate graph of double star $DS_{n,n}$ is a prime graph.

Theorem 3.5 The duplicate graph of comb graph Cb_n for all integers $n \geq 2$ is a prime graph.

Proof

Let $V(Cb_n) = \{v_i / 1 \leq i \leq 2n\}$

$E(Cb_n) = \{v_i v_{i+1} / i = 1,3,...2n-1\} \cup \{v_i v_{i+2} / i = 1,3,5,...,2n-3\}$.

Let G be the duplicate graph of comb graph Cb_n for all integers $n \geq 2$.

Now $v_1, v_2, ..., v_{2n}$, $v_1^1, v_2^1, ..., v_{2n}^1$ and $e_1, e_2, ..., e_{2m-1}$, $e_1^1, e_2^1, ..., e_{2m-1}^1$ respectively be the new set of vertices and edgesof the duplicate graph of comb Cb_n.

Then $V(G) = \{v_i, v_i^1 / 1 \leq i \leq 2n\}$ and

$E(G) = \{v_i v_{i+1}^1 / i = 1,3,5,...,2n-1\} \cup \{v_i v_{i+2}^1 / i = 1,3,5,...,2n-3\} \cup \{v_i v_{i-2}^1 / i = 3,5,7,...2n-1\}$

$\cup \{v_i v_{i-1}^1 / i = 2,4,6,...,2n\}$

Now $|V(G)| = 4n$ and $|E(G)| = 4n-2$.

Define the labeling $f: V(G) \rightarrow \{1,2,3,...,4n\}$ as follows

$$f(v_{4i-3}) = 4i-3 \quad \text{if } i = \begin{cases} 1,2,3,\ldots,\frac{n}{2} \ \ for\ n\ is\ even \\ or \\ 1,2,3,\ldots,\frac{n+1}{2} \end{cases}$$

$$f(v_{4i-2}) = 2(2i+n-1) \quad \text{if } i = \begin{cases} 1,2,3,\ldots,\frac{n}{2} \ \ for\ n\ is\ even \\ or \\ 1,2,3,\ldots,\frac{n+1}{2} \end{cases}$$

$$f(v_{4i-1}) = 4i+2n-1 \quad \text{if } i = \begin{cases} 1,2,3,\ldots,\frac{n}{2} \ \ for\ n\ is\ even \\ or \\ 1,2,3,\ldots,\frac{n-1}{2} \end{cases}$$

$$f(v_{4i}) = 4i \quad \text{if } i = \begin{cases} 1,2,3,\ldots,\frac{n}{2} \ \ for\ n\ is\ even \\ or \\ 1,2,3,\ldots,\frac{n-1}{2} \end{cases}$$

$$f(v^1_{4i-3}) = 4i+2n-3 \quad \text{if } i = \begin{cases} 1,2,3,\ldots,\frac{n}{2} \ \ for\ n\ is\ even \\ or \\ 1,2,3,\ldots,\frac{n+1}{2} \end{cases}$$

$$f(v^1_{4i-2}) = 4i-2 \quad \text{if } i = \begin{cases} 1,2,3,\ldots,\frac{n}{2} \ \ for\ n\ is\ even \\ or \\ 1,2,3,\ldots,\frac{n+1}{2} \end{cases}$$

$$f(v^1_{4i-1}) = 4i-1 \quad \text{if } i = \begin{cases} 1,2,3,\ldots,\frac{n}{2} \ \ for\ n\ is\ even \\ or \\ 1,2,3,\ldots,\frac{n-1}{2} \end{cases}$$

$$f(v^1_{4i}) = 4i+2n \quad \text{if } i = \begin{cases} 1,2,3,\ldots,\frac{n}{2} \ \ for\ n\ is\ even \\ or \\ 1,2,3,\ldots,\frac{n-1}{2} \end{cases}$$

Clearly all the labels are distinct.

For any edge $e = v_{4i-3}v^1_{4i-2} \in G$,

$$\gcd(f(v_{4i-3}), f(v^1_{4i-2})) = \gcd(4i-3, 4i-2) \quad \text{if } i = \begin{cases} 1,2,3,\ldots,\frac{n}{2} \ for\ n\ is\ even \\ or \\ 1,2,3,\ldots,\frac{n+1}{2} \end{cases}$$

$=1$ Since the vertex labels are consecutive integers.

For any edge $e = v_{4i-3}v^1_{4i-1} \in G$,

$$\gcd(f(v_{4i-3}), f(v^1_{4i-1})) = \gcd(4i-3, 4i-1) \quad \text{if } i = \begin{cases} 1,2,3,\ldots,\frac{n}{2} \ for\ n\ is\ even \\ or \\ 1,2,3,\ldots,\frac{n-1}{2} \end{cases}$$

$=1$ Since these are consecutive odd integers.

For any edge $e = v_{4i-3}v^1_{4i-5} \in G$,

$$\gcd(f(v_{4i-3}), f(v^1_{4i-5})) = \gcd(4i-3, 4i-5) \quad \text{if } i = \begin{cases} 2,3,4,\ldots,\frac{n}{2} \ for\ n\ is\ even \\ or \\ 2,3,4,\ldots,\frac{n+1}{2} \end{cases}$$

$=1$ Since these are also consecutive odd numbers.

For any edge $e = v_{4i-2}v^1_{4i-3} \in G$,

$$\gcd(f(v_{4i-2}), f(v^1_{4i-3})) = \gcd(2\text{n}+4i-2, 2\text{n}+4i-3) \quad \text{if } i = \begin{cases} 1,2,3,\ldots,\frac{n}{2} \ for\ n\ is\ even \\ or \\ 1,2,3,\ldots,\frac{n+1}{2} \end{cases}$$

$=1$ Since these are consecutive integers.

For any edge $e = v_{4i-1}v^1_{4i-3} \in G$,

$$\gcd(f(v_{4i-1}), f(v^1_{4i-3})) = \gcd(2\text{n}+4i-1, 2\text{n}+4i-3) \quad \text{if } i = \begin{cases} 1,2,3,\ldots,\frac{n}{2} \ for\ n\ is\ even \\ or \\ 1,2,3,\ldots,\frac{n-1}{2} \end{cases}$$

$=1$ Since the labels are consecutive odd integers.

For any edge $e = v_{4i-1}v^1_{4i} \in G$,

$$\gcd(f(v_{4i-1}), f(v^1_{4i})) = \gcd(2\text{n}+4i-1, 2\text{n}+4i) \quad \text{if } i = \begin{cases} 1,2,3,\ldots,\frac{n}{2} \ for\ n\ is\ even \\ or \\ 1,2,3,\ldots,\frac{n-1}{2} \end{cases}$$

$=1$ Since these are consecutive integers.

For any edge $e = v_{4i-1}v^1_{4i+1} \in G$,

$\gcd(f(v_{4i-1}), f(v^1_{4i+1})) = \gcd(2n+4i-1, 2n+4i+1)$ if $i = \begin{cases} 1,2,3,\ldots,\frac{n-2}{2} \text{ for } n \text{ is even} \\ or \\ 1,2,3,\ldots,\frac{n-1}{2} \end{cases}$

$=1$ Since the vertex labels are consecutive odd integers.

And for any edge $e = v_{4i}v^1_{4i-1} \in G$,

$\gcd(f(v_{4i}), f(v^1_{4i-1})) = \gcd(4i, 4i-1)$ if $i = \begin{cases} 1,2,3,\ldots,\frac{n}{2} \text{ for } n \text{ is even} \\ or \\ 1,2,3,\ldots,\frac{n-1}{2} \end{cases}$

$=1$ Since these are all consecutive integers.

Thus f admits prime labeling.

Hence G is prime graph.

Theorem 3.6 The duplicate graph of the bistar graph $B_{n,n}$ for all integers $n \geq 2$ is a prime graph.

Proof

Let $V(B_{n,n}) = \{v_0, u_0, v_i \,/\, 1 \leq i \leq 2n\}$

$E(B_{n,n}) = \{v_0v_i \,/\, 1 \leq i \leq n\} \cup \{u_0v_i \,/\, n+1 \leq i \leq 2n\} \cup \{u_0v_0\}$

Let G be the duplicate graph of bistar $B_{n,n}$ for all integers $n \geq 2$.

Now let $v_0, u_0, v_1, v_2, \ldots, v_{2n}, v^1_0, u^1_0, v^1_1, v^1_2, \ldots, v^1_{2n}$ and $e_1, e_2, \ldots, e_{2n+1}, e^1_1, e^1_2, \ldots, e^1_{2n+1}$ respectively be the new set of vertices and edges of the duplicate graph of bistar $B_{n,n}$

Then $V(G) = \{u_0, u^1_0, v_i, v^1_i \,/\, 0 \leq i \leq 2n\}$

and $E(G) = \{v_0v^1_i \,/\, 1 \leq i \leq n\} \cup \{v^1_0v_i \,/\, 1 \leq i \leq n\} \cup \{u_0v^1_i \,/\, n+1 \leq i \leq 2n\} \cup \{u^1_0v_i \,/\, n+1 \leq i \leq 2n\} \cup \{v_0u^1_0\} \cup \{v^1_0u_0\}$

Here $|V(G)| = 4n+4$ and $|E(G)| = 4n+2$.

Case (i)

Define the labeling $f : V(G) \to \{1,2,3,\ldots,4n+4\}$ as follows

$f(v_i) = 2i+1$ if $i = 0,1,2,\ldots n$

$f(u_0) = p$

where $'p'$ is the largest prime less than or equal to $4(n+1)$.

According to Bertrands Postulate such a prime exist with $2(n+1) < p < 4(n+1)$

. $f(v_i) = 2i+1$ if $i = n+1, n+2, n+3, \ldots \frac{p-3}{2}$

$f(v_i) = 2i+3$ if $i = \frac{p-1}{2}, \frac{p+1}{2}, \ldots .2n$

$f(v^1_0) = 2$

$f(u_0^1) = 4$

$f(v_i^1) = 2i + 4$ if $i = 1, 2, \ldots 2n$

Case(ii)

If $p = 4n + 3$ then we define the labeling $f : V(G) \to \{1, 2, 3, \ldots, 4n + 4\}$ as follows

$f(u_0) = p$

and $f(v_i) = 2i + 1$ if $i = 0, 1, 2, \ldots 2n$

$f(v_0^1) = 2$

$f(u_0^1) = 4$

$f(v_i^1) = 2i + 4$ if $i = 1, 2, \ldots 2n$

Clearly all the labels are distinct.

For any edge $e = v_0 v_i^1 \in G$,

$\gcd(f(v_0), f(v_i^1) = \gcd(1, 2i + 4)$ if $i = 1, 2, \ldots n$

$= 1$

For any edge $e = v_0 u_0^1 \in G,$

$\gcd(f(v_0), f(u_0^1)) = \gcd(1, 4) = 1$

For any edge $e = v_0^1 v_i \in G,$

$\gcd(f(v_0^1), f(v_i)) = \gcd(2, 2i + 1)$ if $i = 1, 2, \ldots n$

$= \gcd(2, odd) = 1$

For any edge $e = v_0^1 u_0 \in G,$

$\gcd(f(v_0^1), f(u_0)) = \gcd(2, p) = 1$

For any edge $e = u_0 v_i^1 \in G,$

$\gcd(f(u_0), f(v_i^1)) = \gcd(p, 2i + 4)$ if $i = n + 1, n + 2, n + 3, \ldots 2n$

$= 1$

And for any edge $e = u_0^1 v_i \in G,$

$\gcd(f(u_0^1), f(v_i)) = \gcd(4, f(v_i))$ if $i = n + 1, n + 2, n + 3, \ldots 2n$

$= \gcd(4, odd)$

$= 1.$

Thus f admits prime labeling.

Hence G is prime graph.

IV. CONCLUSION

Here we investigate six corresponding results on prime labeling analogues work can be carried out for other families also.

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